Line Sweep Algorithms – Handout

(by Schalk-Willem Krüger)

1. Closest Pair: Sample Source Code

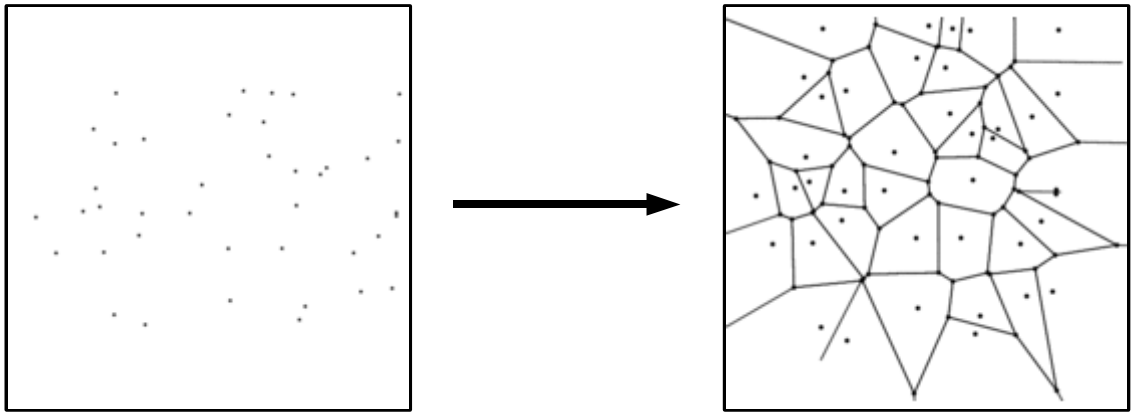
```
#include <stdio.h>
#include <set>
#include <algorithm>
#include <cmath>
using namespace std;
#define px second
#define py first
typedef pair<long long, long long> pairll;
int n;
pairll pnts [100000];
set<pairll> box;
double best;
int compx(pairll a, pairll b) { return a.px<b.px; }
int main () {
    scanf("%d", &n);
    for (int i=0;i<n;++i) scanf("%lld %lld", &pnts[i].px, &pnts[i].py);
    sort(pnts, pnts+n, compx);
    best = 1500000000; // INF
    box.insert(pnts[0]);
    int left = 0;
    for (int i=1;i<n;++i) {
        while (left<i && pnts[i].px-pnts[left].px > best) box.erase(pnts[left++]);
        for (typeof(box.begin()) it=box.lower_bound(make_pair(pnts[i].py-best, pnts[i].px-best));
             it!=box.end() && pnts[i].py+best>=it->py; it++)
                best = min(best, sqrt(pow(pnts[i].py - it->py, 2.0)+pow(pnts[i].px - it->px, 2.0)));
        box.insert(pnts[i]);
    printf("%.2f\n", best);
    }
    return 0;
}
```

2. Union of rectangles: Sample Source Code

```
#include <cstdio>
#include <algorithm>
using namespace std;
struct event {
        int ind;      // Index of rectangle in rects
        bool type;    // Type of event: 0 = Lower-left ; 1 = Upper-right
        event() {};
        event(int ind, int type) : ind(ind), type(type) {};
};
struct point {
        int x, y;
};
int n, e; // n = number of rectangles; e = number of edges
point rects [1000][2];  // Each rectangle consists of 2 points: [0] = lower-left ; [1] = upper-right
event events_v [2000];  // Events of horizontal sweep line
event events_h [2000];  // Events of vertical sweep line
bool compare_x(event a, event b) { return rects[a.ind][a.type].x<rects[b.ind][b.type].x; }
bool compare_y(event a, event b) { return rects[a.ind][a.type].y<rects[b.ind][b.type].y; }
bool in_set [10000];    // Boolean array in place of balanced binary tree (set)
long long area;         // The output: Area of the union
int main() {   /// x -> v; y -> h
        scanf("%d", &n);
        for (int i=0;i<n;++i) {
                scanf("%d %d %d %d", &rects[i][0].x, &rects[i][0].y,   // Lower-left coordinate
                                     &rects[i][1].x, &rects[i][1].y);  // Upper-right coordinate
                events_v[e] = event(i, 0);
                events_h[e++] = event(i, 0);
                events_v[e] = event(i, 1);
                events_h[e++] = event(i, 1);
        }
        sort(events_v, events_v+e, compare_x);
        sort(events_h, events_h+e, compare_y);  // Pre-sort set of horizontal edges
        in_set[events_v[0].ind] = 1;
        for (int i=1;i<e;++i) {  // Vertical sweep line
                event c = events_v[i];
                int cnt = 0;  // Counter to indicate how many rectangles are currently overlapping
                // Delta_x: Distance between current sweep line and previous sweep line
                int delta_x = rects[c.ind][c.type].x - rects[events_v[i-1].ind][events_v[i-1].type].x;
                int begin_y;
                if (delta_x==0) continue;
                for (int j=0;j<e;++j) if (in_set[events_h[j].ind]==1) {  // Horizontal sweep line
                        if (events_h[j].type==0) {
                                if (cnt==0) begin_y = rects[events_h[j].ind][0].y;  // Block starts
                                ++cnt;
                        } else {
                                --cnt;
                                if (cnt==0) { // Block ends
                                        int delta_y = (rects[events_h[j].ind][1].y-begin_y);
                                        area+=delta_x * delta_y;
                                }
                        }
                }
                in_set[c.ind] = (c.type==0);
        }
        printf("%lld\n", area);
        return 0;
}
```

3. Further sample problems

- Voronoi diagram: Given a set of points, divide the space into regions around each point, so that all coordinates in the region are the closest to the point in that region.
  (http://www.cs.sunysb.edu/~algorith/files/voronoi-diagrams.shtml)

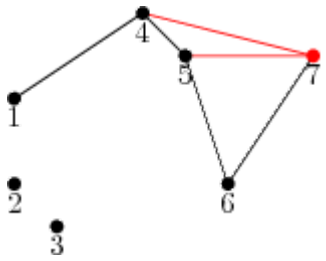


- CultureGrowth: Compute area of the convex hull of a set of points.
  (Topcoder: http://www.topcoder.com/stat?c=problem_statement&pm=3996&rd=7224)
- PowerSupply: http://www.topcoder.com/stat?c=problem_statement&pm=5969
- ConvexPolygons: http://www.topcoder.com/stat?c=problem_statement&pm=4559&rd=7225

4. Convex hull using sweep line

Extracted from http://www.topcoder.com/tc?module=Static&d1=tutorials&d2=lineSweep [by Bruce Merry]:

> *The convex hull of a set of points is the smallest convex polygon that surrounds the entire set, and has a number of practical applications. An efficient method that is often used in contests is the Graham scan [2], which requires a sort by angle. This isn't as easy as it looks at first, since computing the actual angles is expensive and introduces problems with numeric error. A simpler yet equally efficient algorithm is due to Andrew [1], and requires only a sort by X for a line sweep (although Andrew's original paper sorts by Y and has a few optimizations I won't discuss here).*
>
> *Andrew's algorithm splits the convex hull into two parts, the upper and lower hull. Usually these meet at the ends, but if more than one points has minimal (or maximal) X coordinate, then they are joined by a vertical line segment. We'll describe just how to construct the upper hull; the lower hull can be constructed in similar fashion, and in fact can be built in the same loop.*
>
> *To build the upper hull, we start with the point with minimal X coordinate, breaking ties by taking the largest Y coordinate. After this, points are added in order of X coordinate (always taking the largest Y value when multiple points have the same X value). Of course, sometimes this will cause the hull to become concave instead of convex:*



> *The black path shows the current hull. After adding point 7, we check whether the last triangle (5, 6, 7) is convex. In this case it isn't, so we delete the second-last point, namely 6. The process is repeated until a convex triangle is found. In this case we also examine (4, 5, 7) and delete 5 before examining (1, 4, 7) and finding that it is convex, before proceeding to the next point. This is essentially the same procedure that is used in the Graham scan, but proceeding in order of X coordinate rather than in order of the angle made with the starting point. It may at first appear that this process is $O(N^2)$ because of the inner backtracking loop, but since no point can be deleted more than once it is in fact O(N). The algorithm over-all is O(N log N), because the points must initially be sorted by X coordinate.*